Effect of Variation of Penstock Parameter on Mechanical Power

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***Abstract*-Study of hydraulic transients in hydropower plants especially in hydraulic turbine units with penstock is an important aspect. These transients cause large pressure and sub pressure oscillations in turbine hydraulic system. Hence these must be evaluated to avoid mechanical failure. In this paper, a turbine penstock model has been analyzed to study the effect of water hammer. Also an attempt has been made to analyze the effect of length, diameter and thickness of the penstock on water hammer. The simulation model of penstock turbine system is developed in a MATLAB/Simulink-based software environment. It was observed that with the increase in the length, diameter, and thickness of the penstock, the effect of water hammer on mechanical power would reduce to an extent. It was also observed that with the change in the material used in construction of penstock, the effect is decreased.**

Keywords-Water Hammer; Turbine-penstock Model; Mechanical Power; Mathematical Model

I. INTRODUCTION

The regulating system of hydroelectric power plant is a complex system concerning hydraulic dynamics and mechanic-electric dynamics [1]. The water inertia in the pressure conduit system, the nonlinear characteristics of the hydroelectric generating unit, the nonlinear time-varying characteristics of the transferred coefficients for the hydraulic turbine and the load disturbance of the power system make the plant difficult to be regulated and controlled especially when the system is isolated from the grid. The hydraulic turbine governor plays a very important role in hydroelectric power plant. Its performance affects the quality of power supplied and results in safe and stable operation of hydraulic turbine generating unit. Thus it is necessary to research on the transients of the hydraulic turbine.

The study conducted to minimize the effect of the water hammer on the turbine blades. The hydro turbine has distinguished features: (i) due to water inertia, its column compressibility, and penstock-wall elasticity; (ii) open loop transfer function has a zero located on the right hand s-plane (non-minimum phase system); and (iii) change in water head causes variation of its parameters. The water inertia effect results in turbine flow lag behind the gate opening. And elasticity effect leads to rise of traveling pressure waves on the upstream (known as water hammer phenomenon). Thus, the hydro plant is a nonlinear, *nonstationary* system whose characteristics vary significantly with unpredictable load. Its performance is influenced by the characteristics of the water column feeding the turbine. The water hammer occurrence in penstock in mathematical terminology represents an irrational term. This elastic effect is represented by a delay e^{-s2Te} in the hydraulic structure, where T_e is the penstock elastic time & is given by $\frac{L}{\sqrt{a_g/\alpha}}$.

Effect of water hammer on the turbine blades are due to sudden opening of gates or in other words, due to the pressure exerted by the water flowing through the conduit. The effect of water hammer can be reduced either by increasing the length of the penstock or by using penstock of different materials. In this, the effect has been analyzed by developing the hydraulic turbine penstock transfer function [2, 3, 4].

The MATLAB/Simulink program provides a relatively easy-to-use, versatile, and powerful simulation environment for the nonlinear dynamics researches on hydropower plants. In this paper, the simulation model of penstock turbine system is developed in a MATLAB/Simulink-based software environment. These nonlinear characteristics of hydraulic turbine and the non-elastic water hammer effect of pressure water supply conduit are considered in the modeling.

II. MATHEMATICAL MODEL FOR DYNAMICS OF TURBINE AND PENSTOCK

A typical hydro power plant scheme is shown in Fig. 1. Water from the reservoir enters the tunnel and flows through the penstock before reaching the hydro turbine inlet. Next, it flows into scroll casing, which evenly distributes around the runner blades. The runner is mounted on a common shaft with electric generator. The water flow into the turbine is regulated by means of wicket gates, opened, and closed by an oil-hydraulic servomechanism controlled by the governor. The governor acts upon whenever there is mismatch between the torque developed and electrical demand on the generator [5].

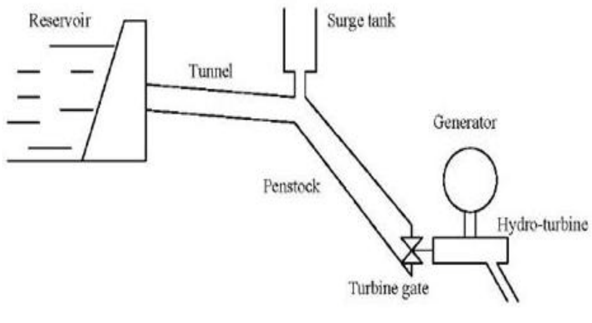

Fig. 1 A general layout of hydro power plant

The linear model of the turbine can be written as follows [3, 6, 7, 8]:

$$\Delta q = a_{11}\Delta h + a_{12}\Delta z + a_{13}\Delta\omega \quad (1)$$

$$\Delta p = a_{21}\Delta h + a_{22}\Delta z + a_{23}\Delta\omega \quad (2)$$

The turbine constants a_{ij} are the partial derivatives of flow (Δq) and power (Δp) with respect to head (Δh), gate position (Δz) and turbine speed ($\Delta\omega$). This aij is dependent on turbine loading and may be evaluated from the characteristic of turbine at the operating point. These values have to be

measured accurately or taken from turbine model tests. The partial with respect to speed, a_{12} and a_{22} are usually negligible.

The basic idea is to define for each hydraulic component an equivalent electric component. In the case of pipe segment, the electric equivalent circuit can be obtained using the momentum and mass conservation equations [9]:

$$\frac{\partial H}{\partial x}+\overbrace{\frac{1}{a_g A}}^{L'}\frac{\partial Q}{\partial t}+\overbrace{\frac{F\left|Q\right|}{2a_g DA^2}}^{R'}Q=0 \tag{3}$$

$$\frac{\partial H}{\partial t}+\underbrace{\frac{a^2}{a_g A}}_{\frac{1}{C'}}\frac{\partial Q}{\partial x}=0 \tag{4}$$

where,

A is the cross sectional area (m2);

F is the friction factor;

D is the diameter of penstock (m);

a is the wave velocity (m/sec);

a_g is acceleration due to gravity.

Equation (3) and (4) can be written as:

$$\frac{\partial H}{\partial x}+L_e\frac{\partial Q}{\partial t}+R_e Q=0 \tag{5}$$

$$\frac{\partial H}{\partial t}+\frac{1}{C_e}\frac{\partial Q}{\partial x}=0 \tag{6}$$

where,

$L'=\dfrac{1}{a_g A}$ (s2/m3), $C'=\dfrac{a^2}{a_g A}$ (m), $R'=\dfrac{F\left|Q\right|}{2a_g DA^2}$

A pipe must always be divided into a series of N elementary pipe segments with the length dx, in which respect it is necessary to reformulate the Equations (3) and (4) by choosing as state variables the piezometric head in the middle of the segment $H_{i+1/2}$ and the input / output flow rates Q_i / Q_{i+1}. The input / output piezometric head H_i / H_{i+1} become boundary conditions for this pipe segment. The first derivatives of H and Q in the middle of the pipe element i can be written according to Fig. 2 [10]:

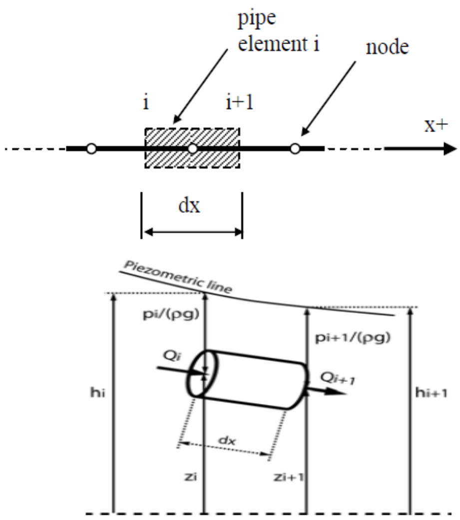

Fig. 2 Discretization of a pipe into n elements

$$\frac{\partial H}{\partial x}\Big|_{i+\frac{1}{2}}=\frac{H_{i+1}-H_i}{dx} \tag{7}$$

$$\frac{\partial Q}{\partial x}\Big|_{i+\frac{1}{2}}=\frac{Q_{i+1}-Q_i}{dx} \tag{8}$$

Using these derivatives, Equations (5) and (6) take the forms:

$$\frac{H_{i+1}-H_i}{dx}+L'\frac{dQ_{i+\frac{1}{2}}}{dt}+R'Q_{i+\frac{1}{2}}=0 \tag{9}$$

$$\frac{dH_{i+\frac{1}{2}}}{dt}+\frac{1}{C'}\frac{\partial Q}{\partial x}=0 \tag{10}$$

with $\quad Q_{i+\frac{1}{2}}=\dfrac{Q_{i+1}+Q_i}{2}$

the Equation (9) and (10) become:

$$C'dx\cdot\frac{dH_{i+\frac{1}{2}}}{dt}=-(Q_{i+1}-Q_i) \tag{11}$$

$$H_{i+1}+\frac{L'dx}{2}\cdot\frac{dQ_{i+1}}{dt}+\frac{R'dx}{2}\cdot Q_{i+1}=H_i-\left(\frac{L'dx}{2}\cdot\frac{dQ}{dt}+\frac{R'dx}{2}\cdot Q_i\right) \tag{12}$$

R = R’.dx; L = L’.dx; C = C’.dx, equations (9) and (10) lead to the electrical equivalent circuit given in Fig. 3 [9, 10].

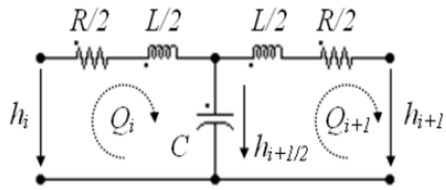

Fig. 3 Electrical equivalent circuit of a pipe segment

The penstock of hydropower plant can be considered as a hydraulic transmission line. This hydraulic transmission line is considered to terminate by using an open circuit at the turbine and short circuit at the reservoir. Equation (11) and (12) may be conveniently solved using Laplace transformation, the solution is given by [11].

The basic hydraulic equations, which determine the flow of a compressible fluid through a uniform elastic pipe, with friction neglected is given by:

$$H_1=H_2\cosh(T_e s)+Q_2 Z_P\sinh(T_e s) \tag{13}$$

$$Q_1=Q_2\cosh(T_e s)+\frac{1}{Z_P}H_2\sinh(T_e s)$$

where, the Subscript 1 and 2 refer to the condition at the end of the conduit of upstream and downstream.

Here in the study, tunnel and surge tank effect have not been considered. The penstock transfer functions relating to the incremental head and flow in terms of complex frequency can be written as follow [3, 8]:

$$\frac{\Delta H(s)}{\Delta Q(s)}=-Z_P\tanh(sT_e+F) \tag{14}$$

This relation depends only on the length of penstock and is independent of turbine's characteristics [8, 10]. An irrational Equation (15) of penstock-turbine with elastic water column effect derived from Equations (1-14) relating to the ratio of incremental torque to changes in guide vane position is given as [9]:

$$\frac{\Delta P_m(s)}{\Delta G(s)} = \frac{a_{23} + (a_{11}a_{23} - a_{21}a_{13})Z_P \tanh(sT_e + F)}{1 + a_{11}Z_P \tanh(sT_e + F)} \quad (15)$$

Assuming an ideal model and having neglected the hydraulic friction losses, (15) can be reduced as:

$$\frac{\Delta P_m(s)}{\Delta G(s)} = \frac{1 - Z_P \tanh(sT_e)}{1 + \frac{1}{2}Z_P \tanh(sT_e)} \quad (16)$$

where,

ΔP_m is the mechanical power generated by the turbine;

ΔG is the gate opening;

Z_P is the normalized hydraulic impedance of penstock & is given by $\frac{T_w}{T_e}$;

T_w is the water starting time in the penstock and is given by $\frac{LQ}{Aa_g H}$;

α is a constant & is given by $\rho a_g \left(\frac{1}{K} + \frac{D}{Ef}\right)$;

L is the length of the penstock (m);

Q is the flow rate in the penstock (m^3/sec);

A is the cross sectional area of the penstock (m^2);

H is the head (m);

a_g is acceleration due to gravity (m/sec^2);

ρ is the density of water (kg/m^3);

K is the Bulk modulus of compressed water (N/m^2);

E is the Young's modulus of elasticity of penstock material (N/m^2);

f is the thickness of the penstock (m).

III. METHODOLOGY

It is difficult to use Equation (15) in its present form for system stability studies. Therefore, for an ideal lossless turbine at full load: a_{11} = 0.5, a_{13} = 1.0, a_{21} = 1.5, a_{23} = 1.0 and F = 0, Equation (15) can be reduced to Equation (16).

It is difficult to use Equation (16) in its present form for system stability studies. It is often helpful to have a finite dimensional approximation. The representation of Equation (16) could alternatively be approximated as lumped parameter equivalent. Expanding the transfer function into a general n^{th}-order model by using the relationship:

$$\tanh(sT_e) = \frac{1 - e^{-2sT_e}}{1 + e^{-2sT_e}} \quad (17)$$

leads to the finite approximation [9]:

$$\tanh(sT_e) = \frac{sT_e \prod_{n=1}^{n=\infty}\left[1 + \left(\frac{sT_e}{n\pi}\right)^2\right]}{\prod_{n=1}^{n=\infty}\left[1 + \left(\frac{2sT_e}{(2n-1)\pi}\right)^2\right]} \quad (18)$$

For n =1 (i.e. with the fundamental component of the water column represented), the Equation (18) is used in Equation (16) to derive the rational transfer function represented as [8, 11]:

$$\frac{\Delta P_m(s)}{\Delta G(s)} = \frac{1 - T_w s + \frac{4}{\pi^2}T_e^2 s^2 - \frac{T_w T_e^2}{\pi^2} s^3}{1 + 0.5T_w s + \frac{4}{\pi^2}T_e^2 s^2 + 0.5\frac{T_w T_e^2}{\pi^2} s^3} \quad (19)$$

To ensure stable frequency regulation under isolated condition, hydro turbine governors are designed to have relatively large transient droop with long resettling time because a change in gate position at the penstock may produce short term power change. The block diagram of a generating unit with hydraulic turbine is shown in Fig. 4.

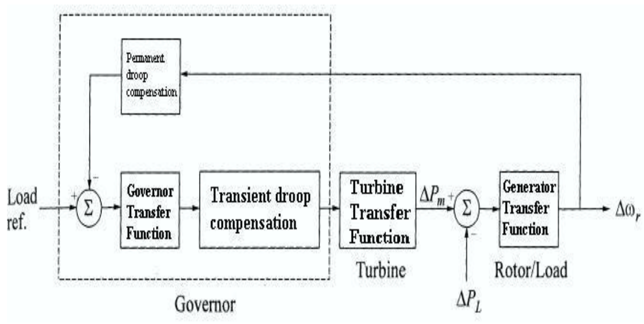

Fig. 4 Block Diagram of a Hydraulic unit

IV. RESULTS AND DISCUSSIONS

The static and dynamic properties of the hydro plant must be known to understand the nonlinear characteristic. A common test to visualize and approximate the nonlinearity in hydro plant characteristic is to test the static behavior of the plant. The static behavior is established by the relationship between the steady-state values of gate position and turbine developed power. The hydraulic turbine generating unit was in standstill and ready to start up as initial condition. The simulation start at first and then the hydraulic turbine generating unit received the signal two seconds later.

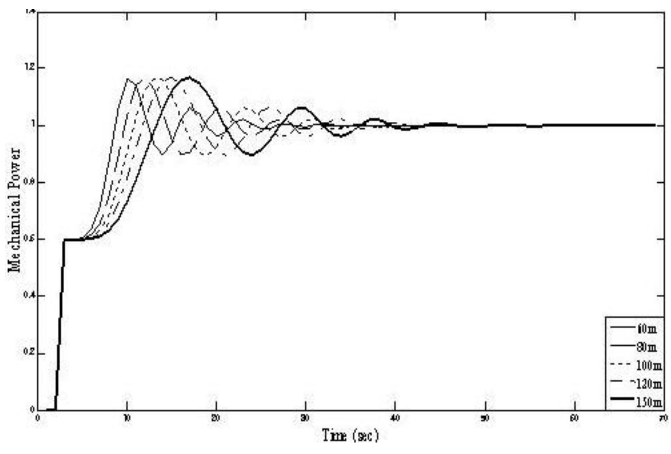

Fig. 5 Effect of Water Hammer on Mechanical Power with change in length of Penstock

Fig. 5 shows the effect of water hammer on mechanical power with change in the length of the penstock. In this situation when the gate is suddenly opened, the water will suddenly flow towards the turbine blades with high pressure. The water with high pressure will strike the blade with a force. This phenomenon is called Water Hammer. Due to water hammer, their will production of pressure wave effects the generation of mechanical power. With the increase in length, the transient developed are less compared with lesser length penstock and the transient damps early.

Fig. 6 shows the effect of water hammer on the mechanical power with change in the diameter of the penstock. From the figure, it is clear that as the diameter of the penstock is increasing, the magnitude of the water hammer effect is getting less, but the time taken to damp the oscillation is becoming more.

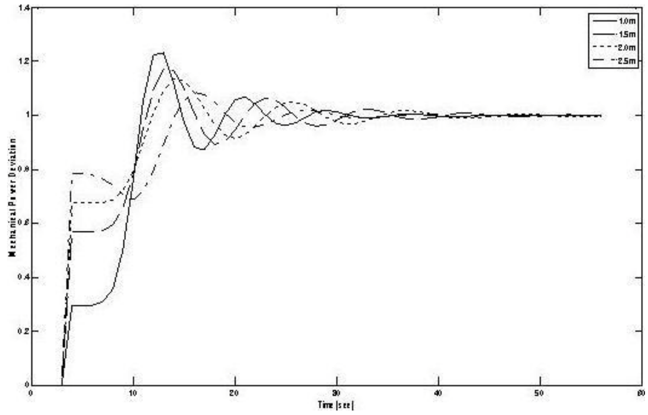

Fig. 6 Effect of Water Hammer on Mechanical Power with change in the Diameter of the Penstock

Fig. 7 shows the effect of water hammer on the mechanical power with change in the thickness of the penstock. With the increase in the thickness, the pressure handling capacity of the penstock increases. The figure shows that with increase in thickness, the oscillations damp quickly. However, the effect of water hammer also increases with the increase in the thickness of the penstock.

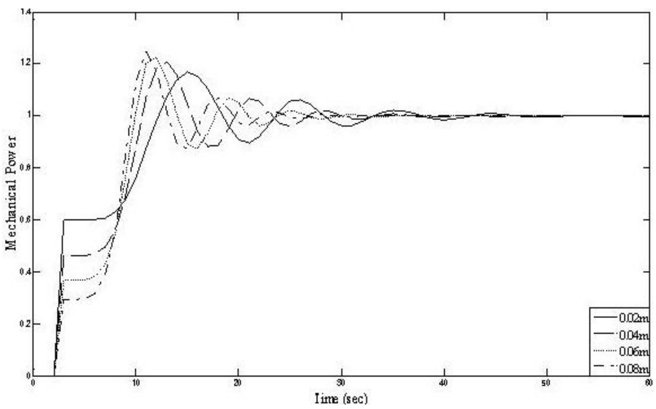

Fig. 7 Effect of Water Hammer on Mechanical Power with change in the Thickness of the Penstock

Fig. 8 shows the effect of water hammer on the mechanical power with change in the material used for the manufacturing of the penstock. The figure shows that penstock manufactured with PVC (polyvinyl chloride) and HDPE (High Density Poly Ethylene) shows fewer transients whereas steel and concrete show transients. But as steel is hard, durable and cheap steel is therefore very often used for manufacturing of penstocks.

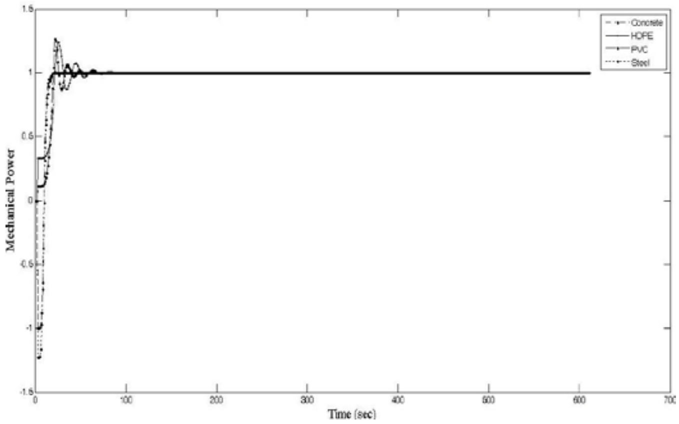

Fig. 8 Effect of Water Hammer on Mechanical Power with change in the material of the Penstock

V. CONCLUSIONS

In the present study, a turbine penstock transfer function has been developed and the analysis of the transfer function has been done with variation in length, diameter, thickness, and material of the penstock. The water column has been considered elastic and water is considered compressible. An electrical equivalent network was also proposed for hydraulic penstock.

In this paper, the simulation model of penstock turbine system has been developed in a MATLAB/Simulink-based software environment. The results have been obtained for different lengths, diameters, and thicknesses of the penstock. These results show that with the small length of the penstock, the transient damp out quickly, whereas with increase in the diameter of the penstock, the magnitude of the transient is lower. It is concluded that for a particular head, an optimal length, diameter, thickness, and material of the penstock must be considered for reducing the effect of the water hammer on the mechanical power.

APPENDIX

Parameters of the system studied are as follows: $R_P = 0.05$, $T_G = 0.2$ sec, $M = 6.0$ sec, $R_T = 0.38$, $T_R = 5.0$ sec, $D = 1.0$, $H = 40$m, $Q = 1.5\ m^3/sec$, $E_{(steel)} = 200\times10^9\ N/m^2$, $E_{(HDPE)} = 0.7\times10^9\ N/m^2$, $E_{(PVC)} = 1.5\times10^9\ N/m^2$, $E_{(Concrete)} = 48\times10^9\ N/m^2$, $K = 2.2\times10^9\ N/m^2$, $\rho = 997.296\ kg/m^3$

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